An LP Based Market Design for Natural Gas

E.G. Read, B. J. Ring, S.R. Starkey, and W. Pepper

Abstract Many electricity markets are now cleared using Linear Programming (LP) formulations that simultaneously determine an optimal dispatch and corresponding nodal prices, for each market dispatch interval. Although natural gas markets have traditionally operated in a very different fashion, the same basic concept can be applied. Since 1999, the Australian state of Victoria has operated a gas market based on an LP approximation to the underlying gas flow optimization problem. Here we discuss market design issues, using a formulation derived from the key gas flow equations. Dual variables on key constraints imply prices which vary by location, as for electricity markets, but also by time. But gas is both delayed and stored within the transportation system itself. This raises a number of operational, pricing, and hedging issues which could be ignored in the case of electricity, but become important when operating this kind of market for gas, or other commodities, such as water, in a supply network where there are delays and storage.

Keywords Linear Programming (LP) • Linearization • Market • Natural gas • Optimization • Pipelines • Prices

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The authors wish to thank AEMO for information used in this chapter, Ray Whaley for assistance with the initial formulation, and Peter Jackson for his input.

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A. Sorokin et al. (eds.), *Handbook of Networks in Power Systems II*, Energy Systems,
DOI 10.1007/978-3-642-23406-4_4,

1 Introduction

Many electricity markets are now cleared using an LP-based "Market Clearing Engine" (MCE) to optimize the value of trade, as determined by participant bids and offers. An MCE formulation typically optimizes power system operations at quite a detailed level, using a detailed representation of the transmission system, and simultaneously optimizing dispatch of generation, transmission power flows, and often ancillary services. This level of detail also enables the MCE to simultaneously determine corresponding prices for energy at each "node" in the transmission system, and often for each ancillary service, in each market dispatch interval.

In 1996, the first market of this type was developed in New Zealand [2]. The Australian "National Electricity Market" (NEM) followed soon after, in 1997, using a re-developed version of the same software, NEM1. That development was also partly built upon the Victorian electricity market, which had been successfully operating in the State of Victoria since 1994, with interconnected trade between Victoria and New South Wales commencing mid 1997 [39]. Broadly similar electricity markets were being discussed or developed in many parts of the world [14], and are now widespread [36]. Thus an extensive literature was developing on topics such as the design of such markets, how market participants might behave in that environment, and how they might hedge their risks over time and space. Hogan [15] had also proposed an auction based model for allocation of gas transport capacity, while McCabe et al. [43] had performed simulations of simple market structures.

These developments lead the Victorian State Government to consider the desirability of developing a similar kind of market for trading natural gas, in that State [42]. The gas system operates in a broadly analogous way to electricity, with a variety of suppliers and consumers, simultaneously injecting gas into, or withdrawing gas from, various points in an interconnected "transmission network". As reported by DPI [10], the Victorian gas system currently has six suppliers, three major retail buyers, and multiple wholesale traders. As shown in Fig. 2, the gas transmission system is a meshed network with multiple inter-regional connectors and active underground gas storages. In 1996, though, there was only one supplier and buyer, operating under a single long term contract, with a simpler network, and no interconnections. The market design discussed here was developed to allow that monopolistic arrangement to be decomposed, and to support evolution towards a more dynamic trading environment.

Convergence towards a consistent market framework also seemed desirable in view of the way in which natural gas interacts with electricity, both as a fuel for generation and a competing supplier of end demand. Thus, one goal of the initial design was to try and create a gas market framework which drew on experience with electricity markets, aligned with the Australian electricity market, and could develop towards greater integration over time. The key question was whether the basic concepts developed for electricity markets could also be applied to gas, in the sense that an analogous "nodal" market-clearing formulation could be developed, and form a practical basis for trading. Further, if that was possible, under what circumstances might it be worthwhile to do so?

This paper focuses on the conceptual market design issues, using an LP formulation analogous to those employed in the electricity sector, based on a representation of the gas system dispatch problem in nonlinear form. In reality, from its inception in March 1999 [1], the Victorian gas market has been dispatched using an MCE based on an LP model developed by ICF International. As described in Pepper et al. [26], the MCE incorporates a number of advanced features to deal with various physical and computational issues, and takes a different approach to linearizing the gas flow equations. That model has proved accurate and reliable, but its fundamental variables are a set of convex weights, which does not make for any intuitive discussion of the forces driving pricing effects in a way likely to prove meaningful to potential developers of gas markets. (In the same way, meaningful discussion of pricing effects in electricity markets must be based on a representation of the fundamental electrical relationships, in a full nodal model, not on the kind of implicit representation some markets use in practice).

The reader is referred to Pepper et al. for a description of the implemented model. Since our goal in this paper is really to discuss the implications of adopting this kind of approach to gas market trading, we abstract away from some of the detail and base our exposition on the original formulation originally developed by Read and Whaley [32], as part of the Putnam Hayes and Bartlett (PHB) team responsible for the market design. Even though this simplified formulation was not implemented in practice, it provides a more accessible introduction to the concepts, starting from a standard "textbook" representation of the gas flow equations, and produces a dual that is more readily interpreted. It will be seen that the resultant formulation is really no more complex than some of the formulations that have been discussed or applied to form electricity markets, particularly if AC power flow equations are modeled [16], and/or ancillary services co-optimized [31], and/or inter-temporal unit commitment constraints represented [17]. Thus this kind of market development does not seem infeasible on grounds of complexity.

The major complication is that the gas flow equations imply that gas is both delayed and stored within the transportation system itself. This raises a number of operational, pricing, and hedging issues which could be ignored in the case of electricity, but become important when operating this kind of market in a gas supply network. One major motivation for discussing these issues, at this time, is that similar issues are likely to be important in markets for other commodities, such as water [28], where delays and storage also occur within the "transportation system" over which the market operates. On the other hand, while Pepper et al. [26] describes a market dispatch process that has proceeded down the path of increasing sophistication and precision, commercial gas market trading is actually based on a highly simplified version of the formulation. In fact, the initial market clearing logic of Ruff [35] only involved a daily clearing of a market in daily gas delivery, while also accounting for overnight storage in the system. For reasons discussed later, the market has still only moved forward to the point of re-clearing to determine prices for the remainder of the day at four-hourly intervals. The studies reported by Frontier Economics [11] did show the potential for significant spatio-temporal differentiation in the marginal value of gas, suggesting the potential value of this kind of market design.

The approach described here was developed in 1997, when there seemed to be little or no literature on the application of optimization models to support, or analyze, trading in gas markets. Since that time, gas market deregulation has proceeded in many places, and the literature has developed accordingly, although to a significantly lesser degree than for electricity markets. Zheng et al. [40] survey gas sector optimization models being applied to optimization of various aspects of gas production, and of gas pipeline network development and operations, but most of those papers do not deal with gas markets, per se. O'Neil et al. [24] and Gabriel et al. [12] model economic equilibrium in gas markets, broadly defined, while Cremer et al. [6] seek to characterize pricing patterns in pipeline networks, with and without a cost recovery requirement. But all these models deal with the issues at a high level, on a much broader scale and longer time frame than envisaged here, and were not intended to form a basis for actual spot gas trading.

De Wolf and Smeers [7], Breton and Zaccour [4], and Gabriel et al. [13] all deal with strategic gaming issues in deregulated gas markets, although only the last models the physical gas transportation system, again on a relatively broad scale. Although the potential for gaming is certainly an important issue for the Victorian system, with its relatively small group of participants, it lies beyond our present scope. Our goal was simply to produce a market framework in which prices are closely aligned with physical system realities, and economic costs. In this respect, the closest approach to ours is probably that of Midthun et al. [21], who discuss a piece-wise linearization approach to modeling the nonlinear pipeline transportation dynamics, and Midthun et al. [21] who apply that approach to consider a gaming problem in which the pipe system operator plays an active role, rather than being a passive "system operator" as in our market paradigm.

2 Market Concepts

Most gas pipelines in the world operate under a contract carriage model. Historical overviews of the emergence of competition in the US may be found in Vany and Walls [37] and Doanne and Spulber [8]. Under this model, the pipeline operator funds its pipeline by selling access to shippers of gas with varying levels of priority. Those with the greatest priority have firm access to the pipeline and tend to pay the most for their access. Those with less priority pay less, but only get access to the pipeline to the extent that it has otherwise unused capacity. Open access regimes may be imposed by regulators who require some transparency to these arrangements, but the basic access arrangement is still via bilateral contracts.

Markets for gas around the world generally operate at hubs between pipelines. Gas can be delivered to these hubs in accordance with pipeline usage contracts, traded at the hub, and hauled away on other pipelines or consumed at the hub. In the US an unregulated natural gas market trades over the New York Mercantile

Exchange (NYME). Futures contracts are traded relative to the principal Henry Hub, in Louisiana. Two equivalent virtual trading point markets are the National Balancing Point (NBP) system in the UK, and the Title Transfer Facility (TTF) in the Netherlands. A Short Term Trading Market (STTM) for gas along these lines operates in Australia at hubs in Adelaide and Sydney, and soon in Brisbane.

The Victorian Gas Market is different in that it operates instead on the concept of "market carriage". The Primary Transmission System (PTS) is funded through Transmission Use of System (TUOS) charges, rather than under bilateral contracting arrangements. This makes it very similar to how electricity transmission systems are funded. With the network funded in this manner, the Victorian Gas Market can operate a "commodity only" market for the trade of gas, and has done so since 1999. Further, with AEMO operating the network, the trade of gas can be used to determine day-to-day transmission access, and (in principle) point-to-point transport charges. Given the obvious analogies between the gas and electricity systems it seems natural to ask whether the concepts that have been applied to design electricity markets might also be applied to design gas markets.

Ignoring a simplified marginal loss adjustment, the Australian electricity market differentiates spot prices by region, so there is only one spot electricity price for all of Victoria. Another point of reference was the New Zealand electricity market, which determines spot prices for each node at which physical injection or off-take occurs [2]. Both operate on the basis of prices for half-hourly trading intervals. Buying and selling (wholesale) electricity is done through a "pool", where electricity generators offer electricity to the marketplace for dispatch through the electricity transmission network. A central market coordinator receives generation offers (and potentially load bids), determines which of those should be accepted (i.e., "clears the market"), implements the optimal dispatch, and announces the corresponding spot prices, all in real time. Thus the goal here was to develop an analogous market design for gas, in which a "market-clearing solution" is determined by an LP optimization model that simultaneously determines:

- A "dispatch" schedule for all gas "injections" and "off-takes" that is optimal, in the sense that it maximize the "value of trade" defined as the benefits delivered to loads (as determined by their bids), minus the costs incurred by suppliers (as determined by their offers).
- A matching set of "nodal" spot prices, varying over time and across network locations, defined by the marginal cost of meeting a (possibly hypothetical) load at each time and location, and applied symmetrically to buy gas from suppliers, and sell gas to consumers.

3 Gas Flow Modeling

In principle, we can divide pipelines up into arbitrarily small cells, and it becomes somewhat arbitrary as to whether primary variables are defined at cell midpoints, or cell boundaries. Our basic nonlinear formulation is based on the initial market design formulation developed by Read and Whaley [32]. They developed their formulation in terms of average midpoint cell values, assuming velocity and pressure changes to be implicitly defined at cell boundaries. That approach was designed to be applied to a fairly discretized representation of a uniform pipeline, though. Pepper et al. [26] model a more general network, approximating over quite long pipeline segments, and modeling nodes where multiple pipes may connect, and various pipe fittings that may induce step changes in velocity and pressure.

We briefly touch on such issues in an Appendix, but here take an intermediate approach, developing a formulation for a single pipeline, with variables primarily defined at cell boundaries, which can be thought of as nodes. The major complication is that natural gas is compressible, unlike some other piped fluids such as water, so its density (and hence pressure) varies, with gas flows being driven by pressure differences. We start with general equations, which allow pressures etc. to differ on each side of each junction, but later assume a single pressure variable at each junction, so as to develop a simplified conceptual formulation. Pepper et al. provide a more detailed development of the actual formulation employed.

The pipeline is split into segments as shown above in Fig. 1. A variable at the centre point of the subscripted *n*th element represents the average value. We discuss flow reversals later but, for simplicity, assume flow is from cell $n-1$, to n, to $n+1$ etc. For simplicity we also assume time periods to be of unit length. The key variables are gas pressure and flow, at the beginning of each pipe segment (or cell), $n = 1,\ldots,N$ and at, or from, the beginning of time period, $t = 1,\ldots,T$. Other variables are derived from these, as necessary. Notationally, constants are represented using normal fonts (e.g. R or H below), whereas variables and indices are represented using italic fonts (e.g. q^i, m_n etc.).

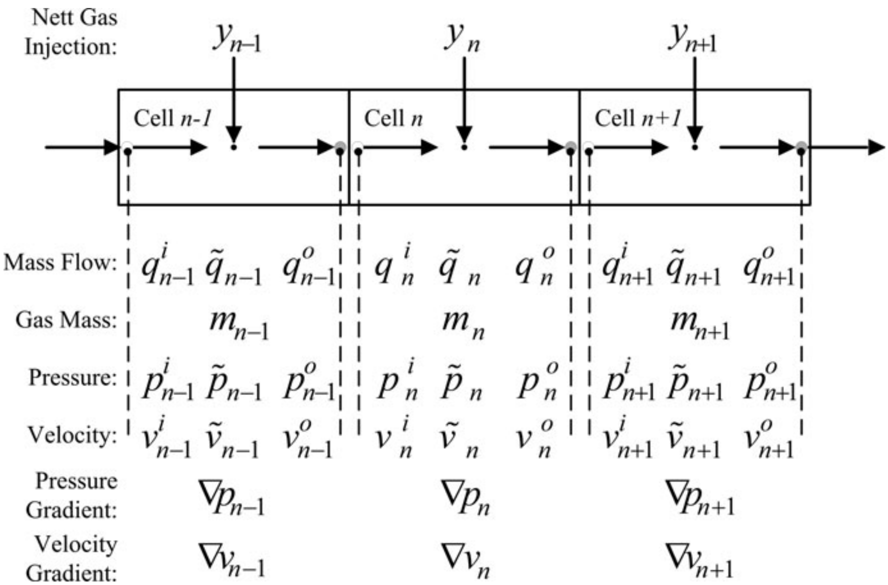

Fig. 1 Key variables for gas pipeline modeling

If there were no compression, the problem could be formulated in terms of fixed volumes of gas travelling from one cell, n, to the next cell, $n + 1$, with some delay. In reality gas density/pressure can vary, and flows are driven by pressure differences but, eventually, balancing forces act to equalize those pressure differences. Gas stored in the transport system is known as "linepack", and this plays an important role in system operation. Daily demands can generally not be met unless linepack storage is built up substantially by pressurizing the gas pipeline overnight. An increase in cell pressure means that more mass is stored in that pipe cell. Thus the mass in the nth cell, at time t, is proportional to the average pressure in the cell, and given by:

$$m_n^t = \mathrm{L}_n \mathrm{G}_n \tilde{p}_n^t \quad (1)$$

$$\tilde{p}_n^t = (p_n^{ti} + p_n^{to})/2 \quad (2)$$

$$\mathrm{G}_n = \mathrm{A}_n/\mathrm{RH} \quad (3)$$

Here the nth pipeline cell has diameter D_n, cross-sectional area A_n, and length L_n. H is the gas temperature , assumed to be constant, and R is known as the "specific gas constant" for the particular gas composition in the pipeline. The rate of gas (mass) flow at any point (e.g. q_n^t) is determined by the pressure of the gas, and its velocity. We will apply this relationship to the midpoint flow/pressure/velocity values, as in (4). This allows us to state the mass conservation equation for each cell, as in (6). Note that, in this equation, injection (y_n^t) is treated interchangeably with mass (m_n^t), and mass is a midpoint value, reflecting average pressure across the cell. Thus injection is implicitly treated as if it were occurring at the midpoint of a cell, increasing pressures at both ends. This is not likely to happen, in practice, but the cell in which injection is assumed to occur can be made arbitrarily short, or represented by a "node" as in Pepper et al.

$$\tilde{q}_n^t = \mathrm{G}_n \tilde{p}_n^t \tilde{v}_n^t \quad (4)$$

$$\tilde{q}_n^t = (q_n^{ti} + q_n^{to})/2 \quad (5)$$

$$m_n^{t+1} = m_n^t + q_n^{ti} - q_n^{to} + y_n^t \quad (6)$$

Modisette and Modisette [22] discuss fluid forces in pipes, initially defining the forces for a single element. By applying the conservation of momentum, they then sum forces and flows across all time periods and pipe elements. We use their results to present what is known as the Bernoulli equation, in its more general form for unsteady flows, which we state for the midpoint velocity/pressure pair:[1]

[1] In this formulation, the pipe is assumed to be horizontal, thus eliminating the gravitational term 'g × sinθ', for an elevation angle of θ. The effect of this force is negligible because natural gas is nearly twice as light as air, at standard conditions. Superscripts i and o are dropped because this equation applies at any point.

$$\Delta v_n^t + \tilde{v}_n^t \nabla v_n^t + \mathrm{B}_n(\tilde{v}_n^t)^2 = -\mathrm{RH}\nabla p_n^t / \tilde{p}_n^t \tag{7}$$

$$\mathrm{B}_n = \mathrm{f} / 2\mathrm{D}_n \tag{8}$$

Here f is the "Moody friction factor", which we assume to be a fixed parameter. This Bernoulli equation describes energies within the pipe system, at a given point in time and space. On the LHS, the equation describes the rate of change of gas velocity in time and then in space, and the final LHS term represents viscous losses. These dynamic velocity terms equate to the RHS proportional pressure change term, i.e. the absolute pressure gradient divided by the actual pressure value. In practice we can make the time periods as short as required to allow the model to solve with sufficient accuracy, and replace derivatives with differences. To allow Eq. 7 to be expressed for the midpoint of a cell, we define:

$$\tilde{v}_n^t = (v_n^{ti} + v_n^{to})/2 \tag{9}$$

$$\Delta v_n^t = v_n^{t+1} - v_n^{t-1} \tag{10}$$

$$\nabla v_n^t = (v_n^{to} - v_n^{ti})/\mathrm{L}_n \tag{11}$$

$$\nabla p_n^t = (p_n^{to} - p_n^{ti})/\mathrm{L}_n \tag{12}$$

Each pipe section will have a working pressure range, and may also have maximum velocity limits. Since flow in a network, especially on loops, can be bi-directional the physical lower bound is likely to have a negative value. Later, we set this lower bound to zero, so as to keep solutions within the range where solutions are convex.[2] In reality, both bounds are more likely to bind at the inlet end of a cell, but for now we impose them on midpoint flows, because this simplifies the dual of our formulation, and discuss variations later:

$$\underline{P}_n^t \leq \tilde{p}_n^{ti} \leq \overline{P}_n^t \tag{13}$$

$$\underline{V}_n^t \leq \tilde{v}_n^{ti} \leq \overline{V}_n^t \tag{14}$$

Equations 1, 2, 4, 5, 6, 7 and 9, 10, 11, 12, 13, 14 describe the key physical gas flow relationships, and consequently feature prominently in the optimization model, with the associated dual variables generating the information required to create consistent market trading prices. Apart from this, an initial pressure (mass) and/flow profile must be assumed, and a target (range) specified for the final period. But the above model is incomplete, because we have not specified how the input variables

[2] Direction of flow is relative to a conventional direction, which for simplicity we define as being from i (inlet) to o (outlet). As with electricity networks this can be generalized by defining a conventional direction for all arcs, then allowing the flow to take a +/− value in that direction.

for each pipe cell link to the outlet variables for the upstream cell. It should be clear that the flow out from cell *n*−*1* to cell *n* equals the flow in to cell *n* from cell *n*−*1*, in mass terms. Both pressure and velocity may change, though, if there is some kind of fitting, or compressor, or just a change in diameter at the junction of two pipelines. A reasonably simple formulation can be produced by assuming that that there will be a proportional change in pressure at such a junction, at least locally around some likely solution level with velocity adjusting to match. But we will simplify further, by assuming that there are no special fittings, or pipeline diameter changes, at junctions. Thus not only mass flow, but pressure and velocity have the same value immediately upstream and downstream from each junction. This means that we can drop the distinction between inlet and outlet variables, with p_n^{ti} etc. just becoming p_n^t etc. and p_n^{to} etc. just becoming p_{n+1}^t etc. Thus, for example:

$$p_n^t = p_n^{ti} = p_{n-1}^{to} \tag{15}$$

4 Basic Market Clearing Formulation

We can now state a formulation for a single pipeline, over a gas trading day. We assume that flow always occurs in a uniform direction, in the direction from cell *n*−*1* to *n* to *n* + *1*, and use the endpoint formulation, assuming pipelines of uniform cross-section, with no abrupt pressure changes due to fittings, as discussed above. Thus G_n becomes simply G from here on. An appendix discusses modeling of complications such as compressors, fittings, and junctions. Initial linepack in the pipeline is inherited from the previous day's end condition, as specified in (20).[3] The end state of the system, at time *T* + *1*, must be set to ensure sufficient linepack carryover; in the required sections of pipeline, to meet next trading day's requirements. In the limit we could try to force final linepack in each cell to a specific value, representing a desired pressure/flow profile for the start of the next trading day, between defined limits. But this is too restrictive, and could lead to extreme price impacts, if not infeasibility, as the system will struggle to meet any exact profile. Still, we may group pipe segments into "zones", $z \in Z$, for which the aggregate end of day linepack must be between defined limits, as in (21).[4]

[3] We could also specify initial flows to get a more accurate representation of the nonlinear equations involved. But that increased accuracy would come at the cost of increasing the likelihood that the LP could not actually find any feasible initial flow/pressure pattern to exactly match the specified parameters.

[4] A combination of upper and lower bounds may suffice to ensure that pressure differentials are also large enough to create sufficient flow in the next trading day. But pressure differential constraints may also be added to ensure this directly, and independently of aggregate linepack levels.

The formulation seeks to maximize the value from allocating gas across time t, and space n, as expressed by Eq. 16. To the extent possible, the optimization balances trade between time periods from $t = 1,\ldots,T$, and over all nodes from $n = 1,\ldots,N$. Participants submit bids and offers as price (Bid_n^t and Offer_n^t) and quantity (x_{di}^t and x_{si}^t) combinations. Individual participant involvement depends on the physical configuration of the system, at each specific location. Many pipe cells will only have extractions, while many others only injection, but most will have neither. For each individual consumer or supplier spot market bidding is represented by Eqs. 16 and 17, where the index subscript i indicates a single bid or offer tranche from a demand *(d)* or supply *(s)* side participant.[5] The aggregate net injection of gas into the system into cell n, at time t, is given by Eq. 18, and upper and lower limits are imposed on this via Eq. 19. Combining all these market bid/offer curves with the gas flow and pressure equations, and ignoring linepack bidding, generates the following nonlinear dispatch formulation.

$$\underset{\substack{x,y,m,p,\tilde{p},q,\\ \tilde{q},\nabla p,v,\tilde{v},\nabla v}}{\text{Maximise}} \sum_t \sum_n \left(\sum_{d \in Dn} \sum_i \text{Bid}_{di}^t x_{di}^t - \sum_{s \in Sn} \sum_i \text{Offer}_{si}^t x_{si}^t \right) \tag{16}$$

Subject to:[6]

Mass, pressure, flow and velocity relationships, within a cell:

$$m_n^t = \text{L}_n \text{G}\, \tilde{p}_n^t \tag{1}$$

$$\tilde{p}_n^t = (p_n^t + p_{n+1}^t)/2 \;:\qquad \psi_n^t \tag{2a}$$

$$\tilde{q}_n^t = \text{G}\, \tilde{p}_n^t \tilde{v}_n^t \tag{4}$$

$$\tilde{q}_n^t = (q_n^t + q_{n+1}^t)/2 \;:\qquad \eta_n^t \tag{5a}$$

$$\tilde{v}_n^t = (v_n^t + v_{n+1}^t)/2 \tag{9a}$$

Mass conservation equation:

$$m_n^{t+1} = m_n^t + q_n^t - q_{n+1}^t + y_n^t \;:\qquad \mu_n^t \tag{6a}$$

Bernoulli energy conservation equation with substitution from (10), over cell:

[5] The notation $i,d \in n$ means i or d is located at node n. $i \in D(n)$ means i is a bid from a demand side participant at n. $i \in d$ means i is a bid/offer from d, etc.

[6] All constraints are $\forall\ n = 1,\ldots,N$ and $t = 1,\ldots,T$ unless otherwise stated. Greek symbols associated with equation numbers indicate the key dual variables which will be significant in later discussion of pricing relationships.

$$\tilde{v}_n^{t+1} - \tilde{v}_n^{t-1} + \tilde{v}_n^t \nabla v_n^t + \mathrm{B}_n (\tilde{v}_n^t)^2 + \mathrm{RH} \nabla p_n^t / \tilde{p}_n^t = 0 \ : \qquad \beta_n^t \tag{7a}$$

Velocity and pressure gradients:

$$\nabla v_n^t = (v_{n+1}^t - v_n^t)/\mathrm{L}_n \tag{11a}$$

$$\nabla p_n^t = (p_{n+1}^t - p_n^t)/\mathrm{L}_n \tag{12a}$$

Pressure bounds:

$$\underline{\mathrm{P}}_n^t \le \tilde{p}_n^t \le \bar{\mathrm{P}}_n^t \ : \qquad \underline{\phi}_n^t ; \bar{\phi}_n^t \tag{13a}$$

Velocity bounds:

$$\underline{\mathrm{V}}_n^t \le \tilde{v}_n^t \le \overline{V}_n^t \ : \qquad \underline{\chi}_n^t ; \bar{\chi}_n^t \tag{14a}$$

Bounds on offer/bid tranches:

$$0 \le x_i^t \le \mathrm{X}_i^t \ : \qquad \underline{\gamma}_i^t ; \bar{\gamma}_i^t \tag{17}$$

Net injection into a cell:

$$y_n^t = \sum_{s \in n} x_s^t - \sum_{d \in n} x_d^t \ : \qquad \lambda_n^t \tag{18}$$

Net injection bounds:

$$\underline{\mathrm{Y}}_n^t \le y_n^t \le \overline{\mathrm{Y}}_n^t \ : \qquad \underline{\delta}_n^t ; \bar{\delta}_n^t \tag{19}$$

Initial linepack status:

$$m_n^0 = M_n^0 \ : \qquad \forall n \in N \tag{20}$$

Final linepack bounds:

$$\underline{\mathrm{M}}_z^{T+1} \le \sum_{n \in z} m_n^{T+1} \le \overline{\mathrm{M}}_z^{T+1} \ : \qquad \forall \ \mathrm{z} \in Z \tag{21}$$

Alternatively, or additionally, terms could be included in the objective function representing a set of net demand curves for linepack in various zones. These would consist of a set of bid tranches, x_{zi}^{T+1} each bidding to buy linepack in zone z, at a price of $\mathrm{PackBid}_{zi}^{T+1}$. Thus (ignoring the possibility that participants might already own linepack rights that they wish to sell) the following could be added to (16) above:

$$+\sum_{z\in Z}\sum_{i}\text{PackBid}_{zi}^{T+1}x_{zi}^{T+1} \tag{22}$$

The total system linepack at the end of the period T, across all nodes n in z, would then have to match the linepack purchased for the final period:

$$m_z^{T+1}=\sum_{n\in z}m_n^{T+1}=\sum_{i\in z}x_{zi}^{T+1} \tag{23}$$

5 Simplification and Linearization

If we are to clear the market using LP software, we must first linearize the nonlinear constraints in the formulation of Sect. 4. Read and Whaley [32] originally proposed to substitute the mass/pressure and flow/velocity relationship in Eqs. 1 and 4 into Eq. 6a, then linearize the resultant nonlinear flow/ pressure equation directly. They also showed how to re-arrange the Bernoulli equation (7a) into two expressions, one involving only velocities, and the other only pressures, and apply Taylor's expansion separately to each.

The relevant derivatives can certainly be formed, but they will not be discussed here, partly because that approach was not actually adopted in practice, and partly because it creates a dual formulation from which it is not particularly easy to deduce pricing relationships. But there were also concerns with respect to the accuracy and convexity of the implied approximation. One proposal was to employ an iterative successive linearization scheme, with each iteration solving an LP linearized around the solution from the previous iteration. A coarse discretization can yield a poor result in this kind of modeling, because small errors can propagate and compound though the equation set. The goal of the representation introduced in Fig. 1 was to re-express the underlying nonlinear differential equations by a set of linear difference equations, with both time and space discretized on a sufficiently fine grid to make the linear assumption reasonable, and to refine the grid further, around a proposed solution, if the results were deemed to be too inaccurate. But this approach is similar to employing Euler's method to solve the underlying differential equations. In practice, that first order method is known to have stability issues, and a fine discretization may be required. Thus higher order methods are generally applied, as described by Dorin and Toma-Leonida [9], for example.

Thus piece-wise linearization within the LP seemed preferable to successive linearization. It was proposed that a piece-wise linear model could be produced using Taylor's expansions, as above, to create "supporting hyper-planes" around a set of points spanning the feasible region. A critical issue, though, was whether the piece-wise linearization so produced would actually form a convex LP feasible region. It can be shown that the flow equations are not actually convex if flows are allowed to reverse, but Read and Whaley argued that an acceptable convex approximation could be found in the vicinity of any likely optimum. There are actually two possible issues here, and a later section discusses why piece-wise linearization may break down in situations where gas, or gas flow, turn out to have negative value. But, concerns about the potential non-convexity of the feasible region itself focused on the first two terms in the Bernoulli equation, relating to velocity changes and kinetic energy, and to the possibility of flow reversal.

Read and Whaley wished to retain these terms in the Bernoulli equation because, at the time, it was unclear whether they would have any significant pricing implications. But almost all other authors, including Pepper et al. [26], have considered those terms small enough to be ignored, on the grounds that a gas, being very light, has little kinetic energy or momentum. Thus it is reasonable to assume that changes to gas injection or withdrawal rates will primarily be reflected in changes to pressure/flow relationships. Friction losses will slow the process, but velocities will quickly respond without significant expenditure of energy to reflect this new "steady state", which then evolves over a longer time frame in accordance with the flow and mass balance equations. Thus most authors use a steady state version of the Bernoulli equation, in which the first (time derivative) term is dropped, and most authors also drop the kinetic energy term. Since the pressure in (7a) in the average pressure over that whole cell, which is proportional to the mass in the cell, Eq. (7a) can be simplified to:[7]

$$\tilde{q}_n^t = \left[\sqrt{\mathrm{A}_n/\mathrm{B}_n\mathrm{L}_n}\right] \times \sqrt{-\nabla p_n^t m_n^t} \; : \qquad \beta_n^t \tag{7b}$$

This form of the equation clearly encloses a convex feasible region.[8] In fact it forms a convex cone, being linear in m_n^t along any ray where $-m_n^t/\nabla p_n^t$ is constant. Although we can now drop (11a), defining ∇v_n^t, because it no longer appears in (7b), $\tilde{v}_n^t$ itself is still defined by (4) and appears in the velocity bounds (14a). But $\tilde{v}_n^t$ can be eliminated from the formulation, along with (4) and (9a), by substituting (4) into (14a) and re-arranging to express those bounds as constraints on flow, as a function of pressure.[9]

$$\mathrm{G}\underline{\mathrm{V}}_n^t \tilde{p}_n^t \leq \tilde{q}_n^t \leq \mathrm{G}\mathrm{V}_n^t \tilde{p}_n^t \; : \qquad {}^{-}\chi_n^t; \bar{\chi}_n^t \tag{14b}$$

[7] A substitution for velocity in terms of flow and pressure is made from (4) and the equation is re-arranged in terms of q with coefficients grouped. This steady sate equation assumes that the mass flow rate is uniform across cell n, and period t. While this approximation is commonly employed, it is not quite consistent with (4), which allows the mass in the cell to change, implying different flow rates at each end.

[8] m_n^t is always positive and (12a) ensures that $-\nabla p_n^t$ is positive, since we are excluding solutions where flows reverse. Thus the RHS is just a constant times their geometric mean which is known to enclose a convex set [3].

[9] Note that, if the lower velocity limit is only used to prevent flow reversal, $\underline{\mathrm{V}}_n^t$ will be set to zero, so the lower bound on mass flow is also set to zero, irrespective of pressure.

If we use (1) to substitute for m in terms of endpoint pressures, p_n^t and p_{n+1}^t, while also substituting for ∇p_n^t from (12a) (which can then be dropped from the formulation) we get:

$$\tilde{q}_n^t = \left[\mathrm{A}_n/\sqrt{2\mathrm{B}_n\mathrm{RHL}_n}\right] \times \sqrt{\left((p_n^t)^2 - (p_{n+1}^t)^2\right)} \quad : \qquad \beta_n^t \tag{7c}$$

This is just the Weymouth equation employed by many other authors, in various forms· If we let σ be the ratio of the downstream and upstream pressures, then substituting σp_{n+1}^t for p_n^t makes it clear that this expression is linear in upstream pressure (or downstream pressure) along any ray from the origin, in the upstream/downstream pressure plane, over the range of interest. (That is for $0 < \sigma < 1$, since otherwise downstream pressure would be higher than upstream pressure, causing flow to reverse.) Over that range, this expression also forms part of a convex cone, as discussed by Tomasgard et al. [38] and Midthun et al. [21].

Zhou and Adewumi [41] take a different route. They form a steady state version of (7) by dropping the first term with its time derivative, but show how to obtain analytic expressions for flow which account for the kinetic energy term. Rearranging their equation (12), for a horizontal pipe, and re-expressing it in our notation gives a modified form of Eq. (7c).[10]

$$\tilde{q}_n^t = \left[\mathrm{A}_n/\sqrt{2\mathrm{B}_n\mathrm{RHL}_n - \mathrm{RH}\ln\left(\frac{p_{n+1}^t}{p_n^t}\right)^2}\right] \times \sqrt{\left((p_n^t)^2 - (p_{n+1}^t)^2\right)} \quad : \qquad \beta_n^t \tag{7d}$$

This approximation can not hold for outlet pressures (and hence outlet/inlet pressure ratios) near zero, because then the logarithm in the divisor tends to infinity, and the predicted flow falls rapidly to zero. For more realistic outlet/inlet pressure ratios, closer to unity, the logarithmic term is close to zero, and this equation creates only a modest adjustment to the Weymouth formula, implying a slightly greater resistance to flow. Substituting σp_{n+1}^t for p_n^t shows that this, too, is linear in pressure along any ray with constant outlet/inlet pressure ratio, and letting that ratio range from a small value up to 1 also forms part of a convex cone.

Although (7d) provides a convex formulation which accounts for the kinetic energy term, we will use the much more common Weymouth type equation in (7c). Martin et al. [19] discuss a piece-wise linearization that could be applied to either equation, using "convex combinations", in the context of a mixed integer formulation. In Victoria a hybrid approach was adopted, using convex combinations to form a piece-wise linear formulation most of the time, but reverting to successive linearization when required to deal with "convexity issues". The detail may be

[10] Assuming a compressibility of $Z = 1$, using the Specific Gas Constant rather than the Universal Gas Constant, and assessing the pressure drop over a cells length L_n.

found in Pepper et al. [26]. But Tomasgard et al. [38] and Midthun et al. [21] discuss piece-wise linearization using hyper-planes created around a set of points, as suggested by Read and Whaley [32]. Using Maple© (see www.maplesoft.com), and re-arranging provides (7e) as the linearization of (7c) around a point denoted by superscript *. Here constant terms are enclosed in square brackets, and the simplification of F_n^{t*} reflects the fact that the square root term on the top line is just a constant times the expression for $\tilde{q}_n^{t*}$, the flow corresponding to (p_n^{t*}, p_{n+1}^{t*}), while the ( ) in the divisor is just the square of the same term.

$$\tilde{q}_n^t = \mathrm{F}_n^{t*} \times \left(\left[p_n^{t*} \right] p_n^t - \left[p_{n+1}^{t*} \right] p_{n+1}^t \right) \; : \qquad \beta_n^t \tag{7e}$$

Where

$$\mathrm{F}_n^{t*} = \left[\frac{\mathrm{A}_n \times \sqrt{\left((p_n^{t*})^2 - (p_{n+1}^{t*})^2 \right)}}{\sqrt{2\mathrm{B}_n \mathrm{RHL}_n} \times \left((p_n^{t*})^2 - (p_{n+1}^{t*})^2 \right)} \right] = \left[\frac{\mathrm{A}_n^2}{2\mathrm{B}_n \mathrm{RHL}_n \tilde{q}_n^{t*}} \right]$$

We will adopt this linearization approach here because it provides a dual formulation from which pricing relationships can be readily be deduced. In the context of a successive linearization scheme, (7e) can be left in equality form, representing the current linearization about a specific point. To create supporting hyper-planes for piece-wise linearization, though, we need to form several copies of (7e), each linearized around a different point, and treats these as inequalities as follows:

$$\tilde{q}_n^t \le p_n^t \times \left[\mathrm{F}_{nk}^{t*} p_{nk}^{t*} \right] - p_{n+1}^t \times \left[\mathrm{F}_{nk}^{t*} p_{n+1,k}^{t*} \right] : k = 1, \ldots, \mathrm{K} : \qquad \beta_{nk}^t \tag{7f}$$

Thus the final simplified LP formulation consists of equations (2), (5) (6a), (7f), (13a), (14b), and (16, 17, 18, 19, 20, 21).

6 Pricing Implications

Although Cremer et al. [6] present a high level analysis of some pricing relationships, we have not seen any systematic analysis of the kind of price patterns that could arise as a result of modeling gas transport dynamics on the time and distance scales discussed here. As with any LP, a complete dual formulation could be stated, and solution of the primal problem will automatically determine the solution of that dual. But market participants, and market designers, will want to understand how those prices are driven by offers and bids, and the kind of pricing patterns that will be produced. To generate that insight, we focus on the key dual relationships determining the way in which spatio-temporal price information generated by solution of the LP reflects the opportunity costs of having one more unit of natural gas available to supply at any time and place.

As always, there will be one dual pricing constraint for each primal variable, and we can generate that constraint simply by collecting terms and summing them. Thus if variable x_i appears with coefficient a_{ik} in constraint k, then the shadow price on constraint k will appear in the pricing constraint for commodity x_i, with the same coefficient. Also note that if a primal constraint relating to cell n and period t contains primal variables relating to, say, cell $n + 1$ and/or period $t + 1$, then the corresponding primal variables for period n and t must appear in primal constraints relating to cell $n-1$ and/or period $t-1$. So the pricing equation for that primal variable will involve shadow prices computed for those constraints. Ultimately we are really only concerned to price commodities traded in the market, in this case gas injected/extracted, and possibly end-of-day linepack.[11] In other words, ultimately, we are mainly interested in the shadow prices on constraints (18). However, these prices depend on the prices of other (non-traded) commodities, and all will ultimately be determined by what is effectively the solution of a set of simultaneous equations, in the LP solution process. Thus we need to consider some other pricing relationships as well.

First note that for a maximization objective, standard duality theory implies that the shadow prices on $<$ constraints will be positive, while those on $>$ constraints will be negative. We have expressed all upper bounds in our formulation as $<$ constraints, and all lower bounds as $>$ constraints. So the shadow prices on all upper bounds will be positive, while those on all lower bounds will be negative. Thus adding the shadow prices on upper and lower bounds effectively creates a composite shadow price, which will be positive if the upper limit binds, and negative if the lower limit binds. (Similarly. the shadow price on an equality constraint will be positive if it binds as an upper limit and negative if it binds as a lower limit.). With that convention in mind, the pricing equations corresponding to the variables for traded quantities, x^t_{di} and x^t_{si}, and for net injection y^t_n, are:[12]

$$\lambda^t_n = \mathrm{Bid}^t_{di} - \left(\underline{\gamma}^t_{di} + \overline{\gamma}^t_{di}\right) \quad : \qquad x^t_{di} \tag{24a}$$

$$\lambda^t_n = \mathrm{Offer}^t_{si} + \left(\underline{\gamma}^t_{si} + \overline{\gamma}^t_{si}\right) \quad : \qquad x^t_{si} \tag{24b}$$

$$\lambda^t_n = \mu^t_n - \left(\underline{\delta}^t_n + \overline{\delta}^t_n\right) \quad : \qquad y^t_n \tag{25}$$

[11] By way of analogy, an electricity market formulation such as that in Alvey et al. [2] may be used to determine prices for line capacity, and even phase angles, but we really only focus on prices for electricity injected/extracted.

[12] Primal variables are associated with each dual equation, just as dual variables were associated with each primal equation.

These conditions are easily interpreted. First, (24a) implies that the local price, λ, will equal the price for some step of the local bid/offer stack if, and only if, that bid/offer is "marginal" at the optimum. That is, if and only if the market is free to take one more, or one less, unit from that step at the offer/bid price because it is not up against either the upper or lower limit of that step. The whole price system is driven by these (marginal) bid/offer prices. Otherwise, one of the γ prices will be non-zero. For offers, the upper (or lower) limit will bind when the market price, λ, is above (or below) the offer price and (24b) merely shows how $\bar{\gamma}_i^t$ (or $\underline{\gamma}_i^t$) adjust to reflect that difference. For bids to buy gas, the situation is reversed, because a positive buy variable corresponds to a decrease in net supply, and has the opposite coefficient in the objective function.

Second, we can take μ_n^t, the shadow price on the mass conservation constraint (6a) to be the system price for gas injected into the main transmission system at that time and place i.e. y_n^t.[13] From (25), the local price, λ, will equal μ if, but only if, the market is free to take one more, or one less, unit from that location, at the local price, because it is not up against either the upper or lower injection/extraction limit at that location. Otherwise, if multiple participants want to inject (or extract) more gas at n than the bottleneck constraints (19) will accommodate, $\bar{\delta}$ (or $\underline{\delta}$) will be non-zero, and μ will be higher (lower) than λ, in order to throttle injection (extraction) back to the bottleneck capacity limit. Local participants could trade between themselves at the λ price, in order to ration limited injection/extraction capacity, but that local price will not impact on prices anywhere else in the system.

The issue is, though, to determine the system price, μ, for non-marginal locations, where injection/extraction is constrained, or limited by upper/lower offer/bid limits. Fundamentally, the price of a unit of gas at any point in the system, and in time, is determined by the marginal value that gas may have in meeting future requirements (or reducing the need for future supply) at some time and place. In this deterministic market-clearing formulation the gas price will also be the marginal cost of supplying gas to that point in time and space, from whatever sources are marginal. Looking at the issue either way, the value of gas at each time and place must be consistent with prices at adjacent times and places, which must be consistent to prices at times and places adjacent to them, and so on, until

[13] This is a slight simplification, with respect to the original formulation, because injected gas will also have a velocity of its own, and if this was modeled it would have some small impact on the solution of the gas transport equations. Thus, in principle, we could have differing prices for "fast gas" and "slow gas" injected at the same time and place. But no such distinction arises here, because the simplified formulation ignores gas velocity.

This situation is conceptually similar to that with respect to electricity injected or consumed with differing "power factors" in electricity markets. Hogan et al. [16] discuss a regime that would explicitly price the active and reactive components determining power factor. But real electricity markets typically only price and trade active power, using a DC approximation to the power flow equations, while relying on other agreements to control power factor within acceptable limits. We assume the same to be true here, with respect to the setting of injection pressure differentials, and hence velocities.

ultimately the entire price system is driven by a small set of marginal offer/bids. In our simplified model these marginal prices will be for gas bought or sold in some node in some period of the day. (If linepack trading were allowed for in the formulation, the marginal prices be prices for end-of-day linepack, as determined by some Packbid)

Just as for electricity markets, the equations linking all these system prices together are the duals of the equations linking the physical quantities together in the gas transmission system. These are the mass balance equations, the shadow prices on which are the system gas prices, μ, and the Bernoulli equations which determine how gas flows, with shadow prices β. And the key variables, in our simplified LP formulation, are the pressures, p_n^t, which (with volumes fixed) effectively measure the mass of gas available at each point in the system, and q_n^t, measuring the flow rates between adjacent locations. Thus we must consider the dual equations associated with these variables. We first discuss the impact which simple limits on flows through space and time would be expected to have, as in the case in a market for stored water, for example. We then discuss how these results will be affected by terms arising from the Bernoulli equation and velocity limits, so as to produce pricing effects which may not be immediately intuitive.

First, gas prices will vary over time. If gas was being stored in a static fashion, like water in a reservoir, then we would have a simple equation linking the price of gas in successive periods to the shadow prices on the upper/lower storage bounds. That is, the price of gas stored in the cell would be the same in each successive period, unless a pressure (i.e. storage) limit was binding. Conversely, an upper (lower) mass/pressure limit would be binding if the price for gas in the next period was higher (lower) than in the current period, giving the system incentives to maximize (minimize) gas carried forward. Note that Eq. 1 is really only a convenience, allowing simplification of some of the equations. Simplistically, each unit of pressure in pipe cell n implies GL_n units of mass there. Thus, dividing by GL_n converts prices associated with pressure in cell n, ($\underline{\phi}_n^t$ and $\bar{\phi}_n^t$), to be compatible with prices associated with mass variables for that cell (μ_n^t from the mass balance equation). So, in this simplified model, the price (μ_n^t) for injected gas (y_n^t) would be inferred from the following equation, describing the way in which those prices evolve over time:

$$\mu_n^t = \mu_n^{t-1} + \left(\underline{\phi}_n^t + \bar{\phi}_n^t\right)/\mathrm{L}_n\mathrm{G} : \qquad \tilde{p}_n^t \tag{26}$$

Second, gas prices will also vary over space. Simplistically, we might expect the price of gas moving through the pipe to be the same in each successive cell, unless a flow limit is binding. And we might expect an upper (lower) flow limit to be binding if (and only if) the price for gas in the next cell is higher (lower) than in the current cell, giving the system incentives to maximize (minimize) gas flowed forward. This would produce inter-nodal pricing impacts analogous to those arising in electricity markets. But we do not have (mass) flow limits, per se, in this formulation, only velocity limits in (14b), and a “friction” term in the simplified Bernoulli (Weymouth) equation (7f) which slows flow, but does not ultimately limit it. And the gas "stored" at one time and place also influences the rate at which gas flows to other places, over time. Thus Eq. 26 is too simplistic. To develop a more accurate representation, of the way in which these pressure relationships affect prices, we first eliminate mass, *m*, from the formulation, by substituting (1) into Eq. 4,[14] to get:

$$\mathrm{L}_n\mathrm{G}\ \tilde{p}_n^{t+1} = \mathrm{L}_n\mathrm{G}\ \tilde{p}_n^t + q_n^t - q_{n+1}^t + y_n^t \ : \qquad \mu_n^t \tag{6b}$$

This leaves us with two sets of pressure/quantity variables in the formulation, one for cell endpoints and one for cell midpoints. In the dual, there will be separate "pricing equations" for each set, but the prices will be linked by the shadow prices on the equations which, in the primal, define the relationships between midpoint and endpoint variables, i.e. (2a) and (5a). Since both midpoint and endpoint variables appear in those equations, their shadow prices appear in pricing equations for both types of variable.

First, inter-locational price interactions are primarily determined by the dual equations for the flow variables. Because the endpoint flow variable, q_n^t, actually appears in the mass balance constraints for cells *n* and *n−1*, and also in the flow averaging equation for both cells (5a), the corresponding dual equation (27) relates the prices on all four of those constraints. But we also have pricing equation (28), for the midpoint flow, ($\tilde{q}_n^t$), which appears in the Bernoulli equation (7f), and the cell velocity bounds (14b). And (28) can be substituted into (27) to give Eq. (29).

$$\mu_n^t = \mu_{n-1}^t + (\eta_{n-1}^t + \eta_n^t)/2 \ : \qquad q_n^t \tag{27}$$

$$\eta_n^t = \left(\sum_k \beta_{nk}^t + \left(\underline{\chi}_n^t + \overline{\chi}_n^t\right)\right) \ : \qquad \tilde{q}_n^t \tag{28}$$

$$\mu_n^t = \mu_{n-1}^t + \left(\sum_k \left(\beta_{n,k}^t + \beta_{n-1,k}^t\right) + \underline{\chi}_{n-1}^t + \overline{\chi}_{n-1}^t + \underline{\chi}_n^t + \overline{\chi}_n^t\right)/2 \ : \quad q_n^t \tag{29}$$

This equation tells us that the price of gas in cell *n* reflects the price of gas in cell *n−1*, upstream, plus the implied cost of moving gas from cell *n−1* into cell *n*. That cost is determined by the shadow prices on constraints limiting flows between adjacent cells. Since the μ_n^t prices are for cell midpoints, the price difference is half determined by conditions in each cell. The friction term in the Bernoulli equation (priced at β_n^t) has a pervasive effect in terms of limiting and slowing inter-cell

[14] And also in Eq. (21), for $t = T + 1$, but that is not relevant here.

flows.[15] The (velocity) flow limits can also play an important role, if binding. If the upper flow limit binds, $\bar{\chi}_n^t$ will be positive, and the value of gas in the downstream cell will be higher than in the upstream cell. But if the lower limit binds (typically at zero) downstream gas will be worth less than upstream gas, and this change can be quite abrupt. This may be a proper reflection of the situation, if reverse flow is physically blocked by a check valve. Otherwise the model solution may not be physically feasible, because there is no way to physically stop flow from reversing, and alternative solutions should be explored, with flow restricted to be in the opposite direction.

Second, inter-temporal price interactions are primarily determined by the dual equations for the pressure variables.[16] Because the endpoint pressure variable, p_n^t, actually appears in the Bernoulli constraints for both cells n and $n-1$, and also in the pressure averaging equation for both cells, the corresponding dual equation (30) relates the prices on all four of those constraints. But we also have a pricing equation for the midpoint pressure, ($\tilde{p}_n^t$), which appears in the pressure averaging equation (2a), and in the pressure and velocity bounds (13a) and (14b) for the midpoint of cell n. So the prices for those constraints appear in the corresponding dual equation (31), which can be re-arranged and re-scaled, to give equation (32), describing the way in which the price of gas (mass) in cell n evolves over time.

$$(\psi_n^t + \psi_{n-1}^t)/2 = \sum_k \left(\left[\mathrm{F}_{n-1,k}^{t*} p_{nk}^{t*} \right] \beta_{n-1,k}^t - \left[\mathrm{F}_{nk}^{t*} p_{nk}^{t*} \right] \beta_{nk}^t \right) \; : \qquad p_n^t \tag{30}$$

$$\psi_n^t = \mathrm{G}\left([\mathrm{L}_n]\mu_n^t - [\mathrm{L}_n]\mu_n^{t-1} + \left[\underline{\mathrm{V}}_n^t \right] \underline{\chi}_n^t + \left[\mathrm{V}_n^t \right] \bar{\chi}_n^t \right) - \left(\underline{\phi}_n^t + \bar{\phi}_n^t \right) \; : \qquad \tilde{p}_n^t \tag{31}$$

$$\mu_n^t = \mu_n^{t-1} + \left\{ \left(\underline{\phi}_n^t + \bar{\phi}_n^t \right) - \mathrm{G}\left(\left[\underline{V}_n^t \right] \underline{\chi}_n^t + \left[\mathrm{V}_n^t \right] \bar{\chi}_n^t \right) + \psi_n^t \right\} / \mathrm{GL}_n \; : \qquad m_n^t \tag{32}$$

The first pair of shadow prices in { } reflects the impact of pressure (storage) bounds in period t, as for the hypothetical simplified model discussed above. The middle pair of terms in { } reflects the fact that having more gas in a cell increases the pressure and hence, for a constant velocity, the rate at which gas (mass) can flow through the cell. If either velocity limit ($\underline{\mathrm{V}}_n^t$,$\mathrm{V}_n^t$) is binding then, it will have a non-zero shadow price ($\underline{\chi}_n^t$,$\bar{\chi}_n^t$). If the lower velocity limit is only used to prevent flow reversal, $\underline{\mathrm{V}}_n^t$ will be set to zero, so that term disappears from this equation, so that constraint plays no role in determining inter-temporal price differentials. Its shadow price, $\underline{\chi}_n^t$, may still contribute to inter-locational price differentials, though, via

[15] As noted earlier, the Zhou and Adewumi [41] equation (7d) effectively implies a small increase to this term, and hence a small increase to inter-spatial differentials, but the pricing effect will be small enough to ignore if the physical impact is small enough to ignore.

[16] In principle, the time derivative terms in the full Bernoulli equation, (7), would create a further inter-temporal link between prices. But we consider that influence to be small enough to ignore if the terms themselves are small enough to ignore.

Eq. (29) above. If the upper velocity limit is binding, the $\bar{\chi}_n^t$ term will reflect a benefit from increasing pressure so as to increase mass flow. But that strategy may be constrained by an upper pressure limit in this cell, in which case the ($\bar{\phi}_n^t$) term associated with the upper pressure limit in Eq. 31 will rise to offset the $\bar{\chi}_n^t$ term here. If the binding pressure limit is in a different cell, its impact will be reflected by the price of gas delivered to cell *n* rising high enough to make any further pressure increase there unattractive.

The last term in { }, ψ_n^t, summarizes the inter-temporal pricing impacts of the Bernoulli equation, as determined by Eq. 30. The RHS of (30) reflects the way in which higher gas pressure at the input end of cell *n*, p_n^t, speeds the flow of gas though cell *n* to cell *n + 1*, while inhibiting the flow of gas through to cell *n* from cell *n−1*, in accordance with the Bernoulli equations for cells *n* and *n−1*. Summation over *k* captures the possibility that more than one supporting hyper-plane from the piece-wise linearization may be binding. The LHS of (30) reflects the fact that higher gas pressure at the input end of cell *n*, increases midpoint pressures in both adjacent cells.

Note that (30) involves prices for two adjacent cells, and we can not simply substitute (30) into (31) to get a complete and explicit expression for the way in which prices in cell *n* evolve over time without any reference to effects in other cells. This reflects the chain-like way in which mid- and end-point variables, and hence prices, are linked along the pipeline. But we can substitute (31) into (30) though, and re-arrange to get:

$$\pi_n^t = \pi_n^{t-1} + \Bigg\{ 2\sum_k \Big(\big[\mathrm{F}_{n-1,k}^{t*} p_{nk}^{t*}\big]\beta_{n-1,k}^t - \big[\mathrm{F}_{nk}^{t*} p_{nk}^{t*}\big]\beta_{nk}^t \Big) - \mathrm{G}\Big(\big[\underline{\mathrm{V}}_n^t\big](\underline{\chi}_n^t + \bar{\chi}_n^t) + \big[\underline{\mathrm{V}}_{n-1}^t\big](\underline{\chi}_{n-1}^t + \bar{\chi}_{n-1}^t) \Big) + \Big(\underline{\phi}_{n-1}^t + \bar{\phi}_{n-1}^t + \underline{\phi}_n^t + \bar{\phi}_n^t \Big) \Bigg\} \; / \; \mathrm{G}[\mathrm{L}_n + \mathrm{L}_{n-1}] \tag{30a}$$

$$\pi_n^t = (\mu_n^t \mathrm{L}_n + \mu_{n-1}^t \mathrm{L}_{n-1})/(\mathrm{L}_n + \mathrm{L}_{n-1}) \tag{33}$$

We can think of π_n^t as the average price for gas in a "pseudo-cell" centered on the boundary between cells *n−1* and *n*, and running from the midpoint of one cell to the midpoint of the next. This price is arguably the correct price for gas injected at a cell boundary, and we can create a cell boundary at any point where gas is to be priced. Then Eq. 30a gives an explicit expression for the way in which the price of gas injected at that point evolves over time, in terms of the impact gas injected there has in both adjacent cells. Other variants can be produced by manipulating the dual equations and/or varying the primal assumptions. For example, we may impose pressure or velocity limits at cell boundaries, rather than at midpoints. But, since the cells modeled can be arbitrarily short, that kind of change does not fundamentally alter the nature of the physical outcomes, or the pricing impact of these equations. The corresponding price terms (ϕ_n^t and χ_n^t) just appear in basically the same form, but in Eqs. 27 and 30 rather than in Eqs. 28 and 31.

Whatever variant of these equations is preferred, they define the key pricing relationships, linking prices over time and space, to create a pattern of gas prices, all driven by marginal offers and bids as discussed previously. The price of gas at any time and place will not only reflect the value that gas will have when it is finally delivered to the location at which it will be consumed, but also the indirect value it may have (positive or negative) in terms of assisting or resisting the flow of gas to other places, at other times, where it may prove to be more, or less valuable. The Bernoulli terms mean that the price may vary from period to period, and from place to place, even if there are no "local" or "immediate" pressure bounds limiting the amount of gas that can be "stored" from one period to the next, and no (absolute) flow bounds limiting the amount of gas that can be "moved" from place to place. All other shadow prices in the dual (including shadow prices on initial and final storage constraints, (20) and (21)), merely adjust to match that pattern.

This situation is analogous to that arising in electricity markets, where a flow constraint on a single link will generate a distinctive "spring washer" price pattern, first described by Ring and Read [33], implying price differentials across all links involved in any loops in which that constraint is involved. These effects arise because power flows according to the laws of physics, splitting across all possible parallel paths in inverse proportion to their impedance. Prices must reflect the fact that some part of any incremental flow will travel over the over-loaded circuit, because it is not possible to "direct" flows to take alternative parallel paths avoiding it. The gas system is similar in that, while valves and compressors give some degree of control over how gas will flow, gas will flow though much of the system entirely according to the laws of physics, not economics. Thus a single binding constraint, at some time and place, will cause difficulty in delivering gas to various downstream locations at, or over, various subsequent periods. Thus it will generate price differentials across space, as in electricity markets, but also across time.

Price differentials could become quite extreme if, as sometimes happens, extreme measures must be taken to keep the system operating. In Victoria, an LNG stockpile is maintained near Melbourne, with stocks being gradually built up over an extended period, so as to be available for release when required in order to maintain pressures when demand is too high to be met by continuous supply through the main pipeline system. The operation of that stockpile is optimized outside the market, as is the operation of other storage facilities, such as the Western Underground Storage Facility (WUGS). This gas is all purchased at market prices, when they are relatively low, then re-sold at times when the marginal value of gas, and hence the optimal gas price, must be very much higher, at the LNG facility. Prices may be even higher at the critical time and place which actually creates the need for such release, if that is not the LNG facility.

In electricity networks, constraint pricing effects are not the only possible drivers of price differentials, though. If transmission system losses are modeled, as in Alvey et al. [2], they will cause pervasive price differentials between all locations, even when no constraints are binding. These transmission system losses are not really analogous to the "friction loss" terms in the Bernoulli equation, though. This is not a loss of gas, but a loss of energy, and its effect is to slow and delay gas delivery, rather than ultimately to limit it. This does represent a potential barrier to the efficient and timely transfer of gas from producers to consumers, but that will only imply inter-locational, or inter-temporal, price differentials in situations where the delay, in combination with insufficient linepack in the right part of the system, forces some other constraints, such as pressure or velocity limits, to bind. When price differentials do occur, these terms, of themselves, typically imply gradual change, as the cumulative effect of the Bernoulli equation on each segment of the pipeline means that flow rates gradually decrease as distance increases.

Compressor operation does imply gas losses, though, and these are accounted for in Eq. 6c, in the Appendix. Thus the price of gas downstream from a compressor must rise in proportion to the marginal gas consumption of the compressor. If achieving the desired pressurization requires a compressor to consume 1% of the gas passing through it, the compressor effectively converts 100 units of upstream gas, at the upstream pressure, to 99 downstream units, at the higher downstream pressure. So, if there were no other costs or constraints involved, they would have the same total value, with the marginal value therefore needing to be (approximately) 1% higher on the downstream side. Abrupt price change can occur at compressors where flow is constrained by a minimum or maximum flow limit, though.

7 An Example

While the equations in the previous section allow us to infer how prices relate in adjacent cells, and periods, we have not made, or seen, any systematic attempt to determine, the variety of system wide price patterns that might emerge. But Annex 3 of Frontier Economics [11] presents some empirical analysis, based on the results produced by the MCE of Pepper et al. [26] for a number of scenarios. That model was developed as a pragmatic replacement for the original conceptual formulation developed by Read and Whaley, and reported here. It ignores the time derivative and kinetic energy terms included in the original formulation but, if these are small, they will also have little impact on prices. While it is linearized in a different way, the pressure/flow relationship employed in this model is essentially the same as that in Eq. 7c above. This approximation has proved sufficient to produce a very good approximation to physical gas flows in the system. Thus we believe the price patterns produced by that the implemented MCE model to be indicative of the kind of price patterns likely to emerge from any implementation of the fundamental market design concept developed here. That is a nodal market based on an LP representation of the underlying network realities, on a short time scale. Here we discuss the price patterns produced for just one of the scenarios considered by Frontier, as reported by Pepper [25].

Figure 2 gives a general locational overview of the Victorian Gas System in terms of main pipelines and nodes. Figure 3, taken from Pepper shows the kind of

Fig. 2 Victorian gas system: network overview

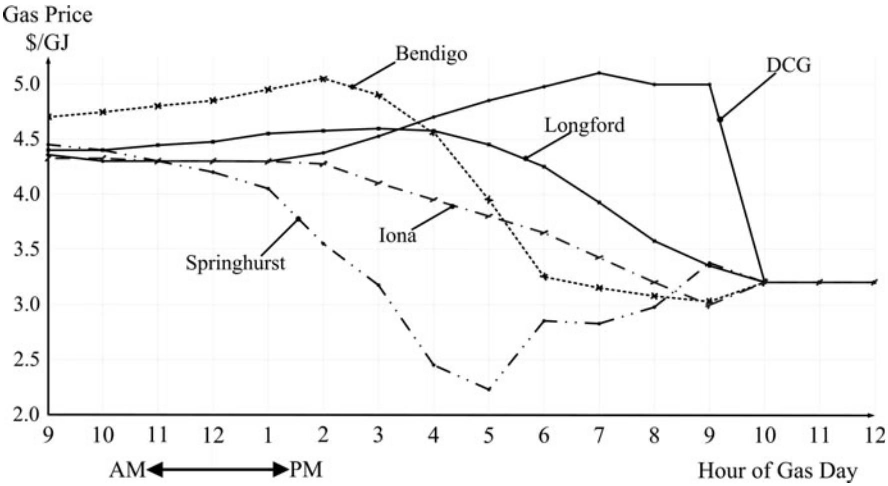

Fig. 3 Victorian gas system example price pattern

price pattern that could occur in this system for a day in which a constraint binds. Pepper notes that prices almost always decline to a flat off-peak value at around 10 PM, with some lag in the outer portions of the system. (Hence, prices after midnight are omitted from Fig. 3). This behavior after the evening peak reflects the fact that the intra-day linepack constraints are no long binding and the MCE only needs to achieve a minimum system-wide linepack constraint by the end of the gas day. If the optimization were changed to a 48 h optimization and the next gas day was also constrained, this flattening of prices may not always occur.

The relatively high prices during this particular gas day stem from two underlying causes. First, the system did not start the day from a position of unconstrained equilibrium, but inherited a gas pressure/flow pattern which made it difficult to meet the day's requirements. This may be seen by the fact that the model assigns differing values to gas in different locations, even at the start of the day. Second, demand for gas during the peak period of the day exceeded the ability of the system to deliver gas from the low cost supply at Longford, which was constrained by production and gas processing plant capacity. Thus, in this solution, pressures were expected to reach minimum allowed levels early in the evening peak period, at Bendigo Junction and at other key points, such as the Dandenong City Gate (DCG), towards the end of the evening peak period. Some higher cost supplies such as LNG or stored gas are thus required to keep pressure at, or above, the minimum pressures.

Thus by 1 PM, the system is clearly struggling to get enough gas through to Melbourne to cover requirements over the rest of the day. So prices rise over the day, then collapse after the critical hour. Prices also rise at Longford, the main injection point, but prices there fall earlier because gas can not reach the critical areas in time to make any difference. Or, more exactly, the value of injecting more gas at that place, in terms of maintaining a pressure differential to increase flow through to the critical area by the end of the critical period falls gradually over several hours as the end of that period approaches.

Iona is, in sense, at the opposite end of the system, at the end of the South-West pipeline. But prices at Iona follow a similar pattern to those at Longford, presumably because Iona also acts as a source from which gas can flow to Melbourne within the critical period. Prices on this pipeline start falling earlier in the day, though, presumably because extra gas at Iona will only have a positive impact on Melbourne delivery if injected early in the day. Bendigo also lies at the end of a pipeline, off the opposite side of the outer pipeline loop from Longford. But Bendigo is not a source, and prices there follow a similar pattern to those at Melbourne. Pepper [25] reported that there is a capacity constraint between Melbourne (DCG) and Bendigo, restricting Bendigo gas availability during the peak period. The Bendigo price drops sharply after 3 PM because changes in supply or demand of gas at Bendigo no longer have much impact on the ability to meet demand at Melbourne through to the end of the evening peak.

This fall below the end-of day value is much more marked for Springhurst, near the neighboring state of New South Wales. There the price drops so early, and so low, as to apparently exhibit almost the opposite of the Melbourne pattern, only rising slowly to match the end-of-day price at the end. Pepper reports that gas is actually flowing north, away from Melbourne, even when prices are higher in Melbourne, at that time. As noted earlier, gas will flow in accordance with the laws of physics and, where there is no valve to control flows, there is no reason why that flow should necessarily enhance economic value. Exactly the same situation arises with respect to loop flows in electricity networks, where counter-price flows are common.

In any case, it is not the price difference at any particular time that determines optimality. The critical issue here is maximizing flow to the major load, at Melbourne, over the critical period. Extra gas in the Northern pipeline, early in the day, is actually assigned a similar value to Melbourne gas because the increased pressure can inhibit northward flows during the critical period, thus allowing the system to meet Melbourne requirements. After 3 PM, though, the value of extra gas there actually falls below the end-of day value. This is because the impact of one unit of additional supply, or reduced demand at Springhurst, requires a reduction in the flow which the model achieves by reducing pressure at the supply end of that pipeline, thus impacting the ability to meet demand on the Northern section of the pipeline.[17]

8 Issues for Market Design and Implementation

8.1 Non-physical Flows and Flow Reversal

The discussion of convexity in Sect. 5 glosses over one significant point which can prove troublesome in implementing this kind of model, whether or not it is used for market clearing. We focused on the convexity of the LP feasible region. But (7c) is an equality, not an inequality, and the actual physical flows are confined to lie exactly on the boundary defined by that equation, rather than within the region bounded by it. As with any nonlinear equation, the set of points it defines can never be convex, even if the equation defines the boundary of a convex set. So the physical feasible region for this problem is definitely not convex. Still, piece-wise linearization is often used to model this kind of situation in LP models, including those of Thomasgard, Midthun and others. This does not create a problem so long as the objective function implies that points on the boundary are preferred to physically infeasible interior points. It will break down, though, if that is not the case.

A similar situation arises when a piece-wise linear representation is used to model quadratic losses for electricity markets, as in Alvey et al. [2]. This creates a convex LP feasible region, and solutions will lie on the appropriate boundary provided losses are economically undesirable. This is almost always the case, but Ring and Read [33] note that a switch must be made from piece-wise linearization

[17] In this case, though, the effect is at least partly due to the fact that this section of pipeline is represented with one pipeline segment. If the representation of the pipeline to Springhurst was divided into multiple pipeline sections, the MCE would be better able to account for the dynamics of the flow relationships and these prices would not fall so far.

to successive linearization if the model determines that the optimal electricity price at some point in the network would be negative. This can actually occur, in situations where increased load (or losses) would relieve pressure on constrained lines in a loop. And that will make it seem desirable for the optimization to propose "solutions" which are not physically feasible, because they imply losses greater than would actually occur, for the specified flow level. Analogous situations could occur here, if the value of having more gas pressure, or a greater pressure differential, becomes negative at some time and place, most likely because it forces gas to flow away from where it is needed, and there are no check valves available to stop that occurring.[18]

In practice, this situation is handled by switching to successive linearization, as discussed by Pepper et al. [26]. But a closely related situation occurs when the model determines that flow reversal would be desirable. We have restricted velocity, and hence pressure differentials to be positive in a defined direction, partly because the Weymouth equation is not convex if extended into the range where the inlet/outlet pressure difference, and hence the flow direction, reverses. This gives us a convex optimization problem, with a unique optimum. There are cases, though, in which a quite different alternative optimum could be considered, with the flow on some pipe segments reversed. Gas could be compressed into a dead-end pipeline segment, for example, and then allowed to flow back out to meet peak demand. And the existence of a ring structure in the DTS suggests that some locations could be supplied sometimes from one direction, and sometimes from the other. In many cases this may not matter, in the sense that the alternative strategies do not greatly affect economic value. But the gas system operator may face some real "integer" choices between significantly different operating strategies.

Ideally, an integer optimization, such as that in Martin et al. [19], could be employed to ensure that the true optimum is found. In reality, the plausible range of operating strategies is quite restricted, at least for this relatively simple system. If the model is observed to force some flows to their lower limits, the operator may make integer decisions with respect to valve and compressor settings, or just with respect to desired flow direction on certain pipeline segments. Given those decisions we can set the limits in (14b) so as to maintain minimum flows in the desired direction, and re-solve using the convex linearization valid for that flow direction but that does mean that the prices determined by the model potentially depend on some high level strategic choices made by the operator, and that may of significant concern, from a participant perspective.[19]

[18] One could imagine this happening in a more extreme case of the Springhurst example above.

[19] It has been suggested, though, that at least some of the observed price effects could have resulted from sub-optimal compressor settings and from inter-temporal constraints on the bids such as overly constrained hourly ramp rates or minimums on hourly injection quantities.

8.2 Rents and Cost Recovery

The existence of price differences, over both space and time, means that there will be a potentially significant "settlement surplus" remaining after all accepted bids and offers have been cleared at the prices produced by the LP optimization. This settlement will be the sum of "rents" collected on all binding constraints. Even when constraints do occur, though, differentials will typically still be small if those constraints can readily be worked around, for example by adjusting compressor settings. Running compressors creates what is effectively only a small loss of gas from the system, and implies equally small differentials, as discussed earlier. If the gas loss factor were constant, this price differential would be just enough to pay for gas losses, thus making no contribution to recovering the cost of the compressor itself. Since the price difference across a compressor reflects marginal losses, and compressor loss functions are convex (see [26]), rents will be generated equal to the price of gas at that point in the network, times the difference between marginal and average losses.[20] But this rent is also small.

Larger price differentials, and hence larger rents, will arise when compressors reach their throughput limits, and/or flows are limited by the other constraints discussed in Sect. 6. As discussed there, a single constraint, binding in a single period, may generate price differences between various locations at various times, and between various times at various locations. Indeed price differences can arise even when no pipeline segment is constrained at all, in terms of absolute flow capacity. So, rent will be collected across a great many links, and periods, where no constraint is binding. This is analogous to the situation in electricity markets, where a single line constraint in a loop will generate price differences, and hence rents on all lines involved in that loop. In both cases, though, the total rent generated by each binding constraint must equal its RHS value times its shadow price.

In the electricity market literature, there has been much debate about the extent to which nodal price differentials, and rents, could or should signal, incentivize, and perhaps fund, transmission network expansion. The desire to signal and incentivize gas network expansion was a significant consideration in developing this gas market framework, too. But we should caution against assuming that rents derived from inter-nodal, or inter-temporal price differences will prove sufficient, of themselves, to fund all optimal network enhancement. If we were to solve a joint operation/expansion optimization, assuming compressor and pipeline, capacity to be continuously expandable with convex costs, we would find an optimum at which the marginal cost of expansion equaled the marginal rent assigned to compressor capacity, by the market clearing prices, on average in NPV terms.

[20] Analogously, a quadratic loss function for electricity transmission implies that marginal losses are always twice average losses, thus generating rents equal to half the loss-induced price differential, even for un-constrained transmission lines.

But neither pipeline nor compressor capacity can be expanded continuously. And, rather than being convex, capacity costs are likely to exhibit significant scale economies, just as for transmission lines. In that case, Read [29, 30] showed that optimal capacity expansion policy implies that the line capacity should be sized so that over its lifetime, it recovered just enough rents to cover the marginal cost of making the line larger, given that a decision had been made to build the line. Put another way, if we approximate transmission capacity costs as having a fixed cost component, plus a variable cost per capacity unit equal to the marginal cost of building more capacity (at the time of construction), then the rents implied by optimal market prices should only recover the "variable" portion, not the "fixed" portion. For transmission lines, Read calculated that this marginal cost component, and hence direct cost recovery from nodal price differentials, was unlikely to be more than 30% of the total cost in an optimally expanded system, while empirical evidence from New Zealand suggested it could be as low as 10% in practice. Rudnick et al. [34] reached similar conclusions. Depending on the strength of scale economies for gas pipeline networks, and compressor equipment similar conclusions are likely to apply. This is not to say that a theoretically optimal transmission expansion/pricing regime could not be driven by these spatio-temporal price differentials. But that regime must rely on forward contracting, prior to expansion, rather than simply on collecting rents from the expanded network, as outlined by Read [30]. In practice, though, such a regime has proved difficult to establish, and supplementary funding, e.g. from industry levies or access charges, is still likely to be required.

8.3 Hedging

One major factor inhibiting further development of the gas market towards a nodal pricing paradigm is that participants fear that they could be exposed to significant price differentials, and not be able to purchase any form of insurance to cover the implied trading risks. Following the electricity market analogy, the development of hedging instruments similar to the "Financial Transmission Rights" (FTRs) developed by Hogan [15] gas been proposed. But a key requirement for an FTR regime to work is that FTRs not be issued beyond what the system is (expected to be) physically capable of delivering. Otherwise, Hogan shows that a revenue adequacy problem arises, because the rents generated on the binding constraints will not be sufficient to support the payments demanded by FTR holders.[21] Conversely, if the flow pattern corresponding to the set of all FTRs held by participants lies within the convex feasible region of the market clearing formulation, the implied "FTR flows"

[21] We do not expect "revenue adequacy" to cover the cost of gas actually consumed by compressors, any more than the cost of actual losses is covered for electricity markets. In both cases these costs must be face by traders as a residual differential, or covered in some other way.

on the lines which turn out to have binding limits in the spot market clearing solution must be no more than their capacity. The rent required to support FTR payments matches the FTR flows times the shadow prices on those constraints, which must be no more than the rent collected as settlement surplus, that as determined by the full RHS capacity of the binding limits, times their respective shadow prices.[22]

The situation is essentially the same for the gas market formulation described here, except that the "transport network" allows gas to be transferred over both time and space, and constraints may thus be on flows from one cell to another, or on one time period to another.[23] Ignoring the possibility of line-pack, what could clearly be supported would be FTRs with a defined delay time, hedging the difference between the gas price in one cell and "start" period and that in another cell in that start period, plus a specific delivery delay. If no constraints bind in such a way as to (directly or indirectly) limit that flow we expect the two prices to be identical. But otherwise, just as for Hogan's electricity model, the rent collected on the flow limiting constraints should suffice to provide hedging for the volume that can be physically transported, with that delay.

Alternatively, we could decompose each "delayed flow" FTR into two components: An instantaneous inter-locational FTR, as in electricity markets, and a locationally specific inter-temporal FTR, hedging between the prices at two different times, for the same location. Ignoring line-pack, neither of these FTR components needs to be physically feasible, on its own. The situation is not really very different from that arising in an electricity market for which FTRs are all expressed with respect to some reference hub. In such a market an FTR from A to B can be decomposed into an "A-to-Hub" component and a "Hub-to-B" component. But the transmission system does not need to be able to support the requested volume of flows from A to the hub, or from the hub to B, only the net flow pattern after all requested flows have been accounted for. In the gas market case we can think of "cell j at time t" as being analogous to a hub. Thus we can define and issue instantaneous inter-locational FTRs, from "cell i at time t" to "cell j at time t",

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[22] This holds even though a single binding constraint may generate price differentials, and hence rents, across all lines involved in any loop in which it is involved. One way to see this is to solve the simultaneous equation system defining power flows in terms of net nodal injections so as to express the line flow directly in terms of net nodal injections. Since a binding constraint holds with equality, the total rent collected on the RHS side of the constraint will be broken down into a set of "nodal rents" on the LHS of the constraint. These nodal rents correspond to the rent collected on that part of a notional flow from the node to a reference node which passes over the constrained line. This representation of the constraint rents making up the settlement surplus can be used to construct constraint based "flow gate rights", as in Chao et al. [5], or classic FTRs, as in Hogan [15].

[23] Convexity issues will arise with respect to "integer" decisions, such as valve or compressor settings, and possibly flow directions. But that is also true with respect to the "integer" decisions, such as breaker or transformer settings, determining the configuration of electricity networks. In both cases any issuer of FTRs must take care to assess the feasibility of supporting those FTRs across the range of network configurations that might apply on the day.

provided we also define and issue locational inter-temporal FTRs from “cell j at time t” to “cell j at time t + delay”. Once issued, such instruments could not be traded independently, but they could be traded using a market clearing optimization that guarantees simultaneous feasibility.

The gas system can support a much wider range of FTRs than this, though, because gas stored as line-pack can typically be released over a wide range of intervals. Thus there is no fixed delay between the time at which gas is injected at i, and the time it is extracted at j. So, for convenience, we could define and issue instantaneous inter-locational FTRs, from “cell i at time t” to “cell j at time t”, and we could also define and issue a wide variety of locational inter-temporal FTRs from “cell j at time t” to “cell j at time r”. Here r may be greater than t, but it could be less than t. In other words it may be possible to extract an incremental unit of gas earlier in the day, provided we know that it will eventually be replaced by a unit injected at time t, and arriving some time later, and that no constraints will actually be violated in the meantime. In particular, instantaneous “trade” will often be possible, even though instantaneous “transport” is not.

The feasible range of such transactions will be limited by binding constraints on pressures or flows, in which case the system will need to incur the costs of re-dispatching other injection, extraction, or compression, in order to make this transaction possible. But the marginal cost of such re-dispatch is exactly what the inter-nodal and inter-temporal price differences measure. And the shadow prices on the binding constraints that determine the inter-nodal and inter-temporal price differences will also generate the rents required to support any simultaneously feasible pattern of inter-nodal and inter-temporal FTRs. More generally, the rents should support any simultaneously feasible pattern of mixed spatio-temporal FTRs, whether or not they are decomposed as in this discussion.

To date, difficulties in conceptualizing hedging arrangements have proven to be a significant deterrent to introducing greater spatio-temporal price differentiation into this relatively small market. But the mathematics of hedging in this kind of gas market seem closely analogous to that in nodal electricity markets. It may seem complex to determine simultaneous feasibility, over both space and time, but all that is required is to notionally clear the proposed FTR trades through a version of the spot market clearing optimization, just as in electricity markets.

9 Experience and Conclusions

In principle, the spatio-temporal prices determined by the formulation discussed in Sect. 6 could be used to coordinate the market at all times, and particularly when congestion creates significant spatio-temporal price differentials. It should be said, though, that the nodal market paradigm described here has not revolutionized markets to anything like the same extent as the analogous electricity market design. This is partly due to inherent differences between the sectors. Valves and compressors make gas flows relatively more controllable than electricity flows, and limit the potential for troublesome "loop flow" effects of the type that at least partially motivated electricity market reform in the US, for example. Thus traditional market paradigms may be relatively more effective in the gas sector than they were in the electricity sector. Nor is there so much need for absolute real-time coordination.

In this particular case, the net volume traded between participants is also not very large, since much of the gas is effectively transported on behalf of vertically integrated participants, who inject their own (contracted) gas at one location, and extract it at another. Still, Pepper et al. [26] describe a detailed LP optimization model that does calculate spatio-temporal prices as above, and the dispatch schedules associated with those prices are used. But they also describe how actual trading prices are determined using a simplified version of the model, in which the gas system is modeled like a simple "tank". That is, gas injected at any location, at any time during the day, is assumed to be able to supply demand at any other point, and time of day. Intra-day price differentials arise because the tank model is re-run several times during the day, but a single gas trading price is calculated each time the model is run for (the remainder of) each trading day.[24]

If no transmission system constraints ever bound, the tank model would always suffice to clear the market. Congestion certainly can occur in Victoria, and give rise to significant pricing effects when it does, as in the example above. The tank model under-estimates the cost of operating the real market at such times, and determines a price which is not consistent with the costs of all suppliers or consumers. Commercially, this is dealt with by "uplift" payments to compensate participants mis-dispatched relative to the daily gas price. But Frontier [11] found that this did not happen often enough to justify moving towards full inter-temporal pricing framework developed here. And we understand that subsequent network developments may have reduced congestion, and averted the need for further market development along these lines.

But the reason the industry has not proceeded further with a more granular market design is definitely not because the experimental evidence suggests that optimal market-clearing prices would always be the same at all times and all locations. On days when constraints bind, price differentials would appear to be of a similar order of magnitude to those found in electricity markets, over both space and time. But that raise a different barrier to further development, because participants are reluctant to expose themselves to the risk implied by potentially significant spatio-temporal price variations that may not well understood, and can not be hedged without development of FTR instruments for which there is no internationally established theoretical framework, or precedent. This concern is

[24] Originally, the market design included end-of-day linepack trading, thus including a version of (22) in the objective function. However, the concept was dropped, due to concerns over price manipulation, and because the feasible end-of-day target linepack range was considered too small and sensitive to be managed by participants.

particularly strong when price variations may be significantly influenced by decisions made by the gas system operator, with respect to compressor settings, etc.

We would argue that the situation is not really very different from that in the electricity sector, where radically different pricing patterns can arise depending on the system operator's decisions about which circuits will operate, and how they will be connected, in each dispatch interval. But there is now significant experience with electricity markets, and protocols have been developed which tend to restrict operator freedom, but deliver benefits to the sector as a whole. A similar process may be expected to evolve in gas markets. Even without full market implementation, the pricing information generated by the model gives a clear measure of the economic costs being imposed by constraints, and the value that might be released by investment in equipment and/or operating practices that could relieve those constraints. But the process of developing appropriate protocols could be controversial and expensive, and possibly not worthwhile in a small gas market such as this.

At this stage, then, the Victorian gas market does not actually employ the full potential of the formulation described here, and the success of the Victorian gas market development, per se, provides only limited evidence with respect to the potential value of an LP-based market-clearing approach. That market has not fully exploited the paradigm's potential, partly due to its small size, degree of vertical integration, and relative lack of congestion. Many markets trade a much greater volume and value of gas than Victoria, though, and congestion seems not uncommon. And, at least in Victoria the lack of any international experience with, or literature on, this type of market structure in the natural gas industry has been a major factor inhibiting further development of a market based on the nodal pricing paradigm. Thus the major intended contribution of this paper is to report that the concepts have actually been developed, tested, and to some extent applied, in the context of a market which has now operated successfully for over a decade. This demonstrates that it is not too difficult to develop a spatio-temporal MCE formulation for a gas market. And experience with that model also reveals the potential for price differences large enough to imply significant potential for economic gains from trading. Thus we consider the paradigm developed here could well prove more fully applicable in larger and more diversified markets, elsewhere.

Just as importantly, there is an increasing interest in the application of so-called "smart" markets [18] to a wide variety of situations. Many of these situations involve storage of some "commodity", such as water e.g. [23, 28], or some form of pollution [27], within a "transportation system", where it may, or may not, be fully or partially controlled by participants and/or in some centralized fashion. This gas market example seems highly relevant to all such developments, because all such markets are likely to exhibit broadly similar spatio-temporal price patterns to those found here, and may need to overcome many of the same conceptual and practical challenges before successful implementation can be expected.

In particular, the way in which stock in transit and/or storage needs to be priced represents a significant step beyond established electricity market practice. And the need to account for the possibility that, as perceptions change, stock will need to be re-priced, perhaps radically, raises significant questions about the validity of deterministic formulations of the type discussed here. Unlike electricity markets we can not rely on participants to manage this in-transit stock, and the relationship between current and future price is determined by the "hard" mathematics of physics and duality, not by trading in futures market reliant on "softer" participant judgments. Thus it may be that market-clearing concepts will eventually have to be developed further, to incorporate stochastic formulations.

Appendix: Modelling Junctions, Fittings, and Compressors

Pepper et al. [26] shows discusses how to deal with several complications ignored in our simplified formulation via simple extensions of the approaches discussed in Sect. 5. Compressors play an important role in many gas systems. By compressing gas at one location they not only allow increased linepack storage, but increase pressure differentials, thus increasing gas flows from one location to another. Obviously, gas compression requires energy input, and in the Victorian system the compressors are themselves powered by gas drawn from the gas transport system. Although compressor fuel use is relatively low, it may be modeled as follows. A gas powered compressor is driven by a proportion of the gas that flows through it; increasing throughput in a pipe requires an increase in the compressor pressure to offset the dynamic losses down the pipe. Increasing Δp necessitates speeding up the compressor and hence increases fuel consumption. For centrifugal gas compressors a quadratic equation relates the change in head (pressure), Δp, volumetric flow at the compressor inlet q_c^i, and impeller speed, RPM, as follows[25]:

$$\Delta p = \mathrm{C}_1 \times (q_c^i)^2 + \mathrm{C}_2 \times q_c^i \times RPM + \mathrm{C}_3 + \mathrm{C}_4 \times RPM^2 \tag{34}$$

We can not use this equation directly in the formulation, though, because RPM is not an LP variable and the equation in this form is nonlinear. Still, we can reasonably assume that compressor operation rules will have been externally optimized and that optimal operation will imply equations giving the minimum gas consumption required to achieve any desired pressure/flow trade-off. Read and Whaley [32] present a number of detailed equations and steps to determine this loss in gas mass, during compressor operation. Basically, if we know the desired flow rate and pressure increase across the compressor, we can calculate the required running speed of the compressor, in RPM, from which we can determine the rate of fuel usage, and hence the actual mass "lost" in the compressor. This loss, which is a function of volumetric flow and pressure change, is the cost associated with

[25] Constants C_1–C_4 are normally stated by compressor manufactures as standard data.

pressurization of the gas in the downstream section of pipeline. So mass conservation must be revised to account for the fuel usage, *Loss*, represented as an effective reduction in mass flow on the discharge side of the compressor:[26]

$$m_n^{t+1} = m_n^t + q_n^{ti} - q_n^{to} + y_n^t - Loss_n^t(q_n^{ti}, \Delta p_n^t) \quad (6c)$$

Ultimately, ignoring commitment of compressor units, this loss is the only specific aspect of compressor operation that needs to be included in the market clearing formulation. The compressor loss term in Eq. 6c) can be reasonably approximated by a convex differentiable function over a convex feasible operating region. Thus it can be linearized as a function of the LP variables, i.e.q_n^t, p_n^t, p_n^{ti} and p_n^{to}. Apart from this, and the fact that pressure rises, rather than falling in the direction of flow, compressors can be treated like other "fittings".

An implementable solution for a real pipe network must also generalize Eq. 6 to represent mass flow balance in situations where multiple inflow pipes of varying diameter and length connect to a similar variety of outflow pipes. Since the mass flow rate is equivalent on each side of "fittings", such as valves, tees and bends, flow through them can be determined by the pressure difference between the two adjacent cells. But constraints and variables may be required to represent pressure drops of specific forms implied by particular fitting types. Some valves basically increase the friction factor in the Bernoulli equation, for a short pipe segment, with a closed valve implying infinite resistance. Pressure reducing valves are designed to reduce pressure to a specific level. This can be enforced by an upper pressure limit, but a slack variable is also required to represent the drop from the upstream pressure level to the specified level. Or a ratio constraint can be used to represent proportional pressure change as may occur when pipelines of different sizes are joined.

For proportional changes, injecting at a junction increases pressure in both adjacent cells, and produces the same kind of pricing patterns. A more detailed formulation, modeling both input and output pressures and assuming constant pressure ratios at boundaries, produces essentially the same pricing equations. In (30), though, the weights on the price terms (β and ψ), for cell $n-1$ now involve the cell boundary pressure ratio. Pressure reducing valves create a pressure discontinuity, though, and a pricing discontinuity can be expected. But, while a more complex formulation may make the pricing relationships more difficult for participants to understand and verify, LP optimization will always ensure that price relationships correctly reflect physical realities, to the extent that they are represented in the LP.

[26] In this simplified representation we are assuming that compressors can be dispatched continuously right down to (near) zero, with no "commitment" costs, penalties, or restrictions. This allows us to form an LP representation with a convex feasible region. In reality there is an integer "unit commitment" problem here, as discussed by Pepper et al. [26].

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